$$-\frac{2e^2-1}{e^2}d[H(e,x)]+3dI=-\frac{e^2-1}{e^2}d[H'(e,x)]-\frac{2e^2-1}{e^2}d[H(e,x)]+3dI.$$

$$\therefore\left[x\sqrt{(e^2x^2-1)(x^2-1)}+\frac{e^2-1}{e^2}H'(e,x)+\frac{2e^2-1}{e^2}H(e,x)\right]_0^1=3I.$$

$$2I=\left[\frac{2}{3}\cdot\frac{e^2-1}{e^2}H'(e,x)+\frac{2}{3}\cdot\frac{2e^2-1}{e^2}H(e,x)\right]_0^1.$$

$$\therefore\quad V=\frac{1568\sqrt{3}\pi a^3}{3e^2}\left[\frac{2e^2-1}{e^2}H(e,x)+\frac{e^2-1}{e^2}H'(e,x)\right]_0^1,\text{ but } e^2=\frac{49}{48},$$

$$\therefore\quad V=\frac{512\sqrt{3}\pi a^3}{49}\left[50H\left(\frac{7}{4\sqrt{3}}\right)+H'\left(\frac{7}{4\sqrt{3}}\right)\right],$$ where $H$ and $H'$ denote the hyperbolic functions corresponding to the elliptic functions $E$ and $F$.

III. **Remarks by Professor J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.**

This curve, called in French "la courbe du diable," is of the middle point of a chord to the equilateral hyperbola $x^2-y^2=2a^2$, the chord being of constant length and equal to seven times the transverse axis $2a\sqrt{2}$. Its equation is found thus: Let $(x_1, y_1)$ and $(x_2, y_2)$ be the extremities of a chord, and $(x, y)$ any point of the curve, then we have the equations: $x^2-y^2=2a^2\ldots(1)$. $x_2^2-y_2^2=2a^2\ldots(2)$, $2x=x_1+x_2\ldots(3)$, $2y=y_1+y_2\ldots(4)$, and $(x_1-x_2)^2+(y_1-y_2)^2=392a^2\ldots(5)$. Subtracting (2) from (1), and considering (3) and (4), we have $(x_1-x_2)x=(y_1-y_2)y$, whence $y_1-y_2=(x_1-x_2)\frac{x}{y}$. Substituting this in (5), we get $x_1-x_2=\frac{14\sqrt{2}.ay}{(x^2+y^2)^{\frac{1}{2}}}$, $y_1-y_2=\frac{14\sqrt{2}.ax}{(x^2-y^2)^{\frac{1}{2}}}$, and combining these with (2) and (3), we get $x_1=x+\frac{7\sqrt{2}.ay}{(x^2+y^2)^{\frac{1}{2}}}$, $y_1=y+\frac{7\sqrt{2}.ax}{(x^2+y^2)^{\frac{1}{2}}}$. Substituting in (1) and simplifying we finally have $y^4-x^4-96a^2y^2+100a^2x^2=0$.

Query: Can any one furnish a reason for the peculiar name of the "devil's curve," or the name which Prof. Matz employes?

Also solved by *Prof. C. W. M. Black*.

## PROBLEMS.

34. **Proposed by GEORGE LILLEY, Ph. D., LL. D., Park School, Portland, Oregon.**

A hare is at $O$, and a hound at $E$, 40 rods east of $O$. They start at the same instant each running with uniform velocity. The hare runs north. The hound runs directly towards the hare and overtakes it at $N$, 320 rods from $O$. How far did the hound run?

35. **Proposed by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pa.**

Water is running into a vessel in the shape of a frustum of a cone (radii up-

per and lower bases 15 inches and 10 inches, respectively, and altitude 20 inches) at the rate of 10 cubic inches per second. When the depth is 8 inches at what rate is it increasing?

# MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

## SOLUTIONS OF PROBLEMS.

14. Proposed by ALFRED HUME, C. E., Sc. D., Professor of Mathematics, University of Mississippi, University P. O., Miss.

"The center of a sphere of radius $C$ moves in a circle of radius $A$ and generates thereby a solid ring, as an anchor-ring: prove that the moment of inertia of this ring about an axis passing through the center of the direct circle and perpendicular to its plane is $\frac{1}{4}\pi^2\delta ac^2(4a^2+3c^2)$."

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If the moment axis be the axis of $z$, the origin being the center of the ring, and the axis of $x$ and $y$ any two diameters at right angles the required moment could be obtained from $\int\int(x^2+y^2)zdxdy$, having the equation to the surface of the ring.

But the following method quoted by Williamson in the Int. Cal., New York Edition, 1884, art. 212 from Townsend is so concise I prefer to give it.

Let $y$, $Y$ be the distances of any point in the meridian section of the sphere from that diameter of the section parallel to the moment axis, and to the moment axis. Then if $dA$ be the element of area of the generating section, the mass of the elementary ring generated by $dA$ is $2\pi\mu YdA$, and the moment of inertia of this ring is $2\pi\mu Y^3dA$.

$$\therefore \text{ the required } M.I.=2\pi\mu\int Y^3dA=2\pi\mu\int(a+y)^3dA$$

$$=2\pi\mu\int(a^3+3a^2y+3ay^2+y^3)dA\ldots.(1).$$

But from theory, $\int ydA=0$, $\int y^3dA=0$, and if $k$ be the radius of gyration of the generating section, $\int y^2dA=Ak^2$: then (1) becomes

$$M.I.=2\pi\mu aA(a^2+3k^2)=2\pi^2\mu ac^2(a^2+\tfrac{3}{4}c^2)$$